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RESEARCH ARTICLE

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An empirical model for durations in stocks

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Abstract This paper considers an extension of the univariate autoregressive conditional duration model to which durations from a second stock are added. The model is empirically used to study duration dependence in four traded stocks, Nordea, Föreningssparbanken, Handelsbanken and SEB A on the Stockholm Stock Exchange. The stocks are all active in the banking sector. It is found that including durations from a second stock may add explanatory power to the univariate model. We also find that spread changes have significant effect for all series.

Keywords Finance · Multivariate · Transaction data · Market microstructure · Granger causality

JEL Classification Numbers C12 · C32 · C41 · G14

1 Introduction

This paper empirically examines the dependence between durations in stocks from the banking sector traded on the Stockholm Stock Exchange. In this context durations are the times between two consecutive transactions in a stock. To empirically capture dependence between durations in stocks the autoregressive conditional duration model (ACD) of Engle and Russell (1998) is extended by adding lagged durations from a second stock to the conditional mean function.

The economic motivation to studying trade intensity or, the time between transactions originates from the information based model of Glosten and Milgrom (1985). In their model they argue that the information set diffuses among informed and uninformed traders. The informed traders have superior information, public

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or private, regarding the true value of the asset and make use of their information advantage as long as it is profitable. Uninformed traders, however, do not have private information concerning the value of the asset and trade mainly for liquidity reasons. To distinguish between informed and uninformed traders Easley and O'Hara (1992) highlights the importance of the trade intensity or, the time between transactions. In their model non-trading, i.e. long durations between transaction, is an indicator of no new information, neither good nor bad. Trading is on the other hand an indicator of new information. Thus, the probability is high that transactions involve informed traders when the observed durations are short.

New available information to the stockmarket may, for example, concern a specific stock, an industry sector or even the whole stockmarket. The stock specific news may, however, not only be relevant for the value of the specific stock. Values of related stocks, e.g., in the same sector, may also be affected. The possibility that the new information discovered by the increased trade intensity in a specific stock also is relevant for related stocks may therefore influence trading in, e.g., stocks in the same sector. Thus, new information may lead to dependence between durations in different stocks.

With the availability of ultra high frequency data (i.e., within the day series where every transaction is registered) standard econometric tools may not be appropriate as transactions are irregularly spaced over time. One way of handling the irregularity is to aggregate data into regularly spaced intervals and apply, e.g., count data models. However, aggregation results in information loss which is not always desirable. Albeit the complication of modelling irregularly spaced data one influential approach for handling the data is the ACD model. It models the duration between transactions and conditions the duration on recent durations and other explanatory variables. Within the ACD framework several extensions and applications of the original Engle and Russell (1998) model have been presented (e.g., Bauwens and Giot 2001).

Models for the dependence between durations in more than one duration series have proven rather complicated, e.g., Engle and Lunde (2003), Russell (1999) and Bauwens and Hautsch (2004). The difficulty is in modelling an expected duration while at the same time accounting for events during the duration. Pairs of durations with the same starting time would be ideal, but are rarely available in transaction data. Both Engle and Lunde (2003) and Russell (1999) advance a model for the dependence between the trade and the quote arrival processes. Bauwens and Hautsch (2004) present a model for the dependence between durations and report results for German stocks. The model of Engle and Lunde (2003) is closely related to the ACD model as it treats one of the duration series as censored and applies the ACD model. In the same line is the model of Mosconi and Olivetti (2001). Russell (1999) and Bauwens and Hautsch (2004) take another approach and instead model the intensities. An application of the model of Russell (1999) for dependences between duration series have been given by Spierdijk et al. (2002). They find positive dependence between stocks active in the same industry sector.

Here, a technically simpler approach to capturing dependence between duration series is suggested. In the ACD model durations from a second stock beyond the focused one are added to the conditional mean function and the result is empirically evaluated. Admittedly, the proposed models are rather ad hoc even though they attempt to capture the main features of the data generating process. However, the empirical evidence of the current type of models may serve a purpose or inspiration for both building a base of empirical evidence and more probabilistic work. Maximum likelihood and conditional least square are suggested as estimators of the unknown parameters. Further, an impulse response function is plotted and Granger causality is studied. Empirical results are presented for 98 trading days in four stocks in the banking sector at the Stockholm Stock Exchange. The stocks are Nordea, Föreningssparbanken, Handelsbanken and SEB.

The outline of the paper is the following. In Sect. 2 we give a brief account of the ACD model and present the extended version. The section ends with details concerning estimation. Section 3 presents the data, while Sect. 4 presents the empirical results. The final section concludes.

2 Model and estimation

In this section we first give a brief account of the ACD model and then proceed to discuss the extension which is aimed at accounting for a second transaction series. The estimation of unknown parameters is also considered.

2.1 Autoregressive conditional duration model

In the ACD model of Engle and Russell (1998) the time between consecutive transactions is modelled. The duration, d_i, is the time between two consecutive transactions at t_{i-1} and t_i, i.e., $d_i = t_i - t_{i-1}$. The conditional expectation of a duration d_i is specified as $E(d_i | d_{i-1}, \ldots, d_1; x) = \theta_i$. The d_i is conditioned on past durations and other explanatory variables x, and θ_i is specified in such a way that $\epsilon_i = d_i/\theta_i$ is independent and identically distributed. The θ_i may be parameterized as

$$\theta_i = \omega + \sum_{j=1}^{p} \alpha_j d_{i-j} + \sum_{j=1}^{q} \beta_j \theta_{i-j} + \pi' x_{i-1}. \tag{1}$$

In (1) θ_i is parameterized with p lagged durations and q lags of conditional durations. This is called an ACD(p,q,x) where p and q are the orders of the lags in the mean function and x_i is a vector of explanatory variables such as volume and spread at time i.

The unconditional mean and variance of an ACD(1,1) are easy to obtain. By assuming that ϵ_i follows a standard exponential distribution, $\pi = 0$ in (1), and that d_i is weakly stationary the mean is

$$E(d_i) = \mu = \frac{\omega}{1 - \alpha - \beta}. \tag{2}$$

The time invariant unconditional mean together with the obvious $d_i > 0$ condition imply the restrictions $\omega > 0$ and $\alpha + \beta < 1$. The unconditional variance of the ACD(1,1) is given by

$$Var(d_i) = \sigma^2 = \mu^2 \times \frac{1 - \beta^2 - 2\alpha\beta}{1 - \beta^2 - 2\alpha\beta - 2\alpha^2}. \tag{3}$$

For finite variance $\beta^2 + 2\alpha\beta + 2\alpha^2 < 1$ must hold.

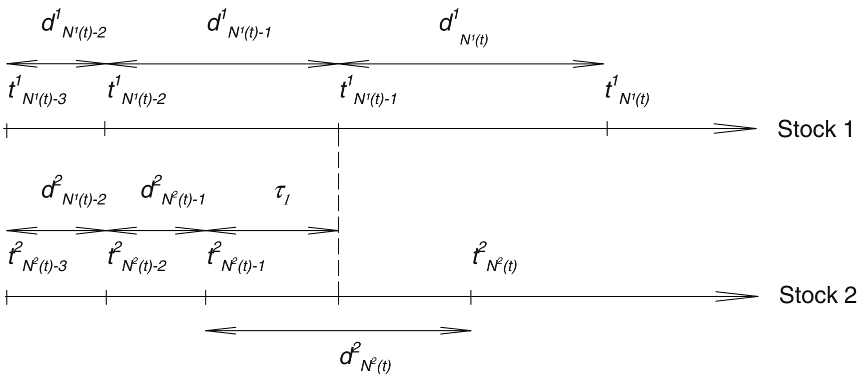

Fig. 1 Illustration of two stocks, stock 1 and stock 2, with transaction times t and durations d. τ_1 is the observed length of the most recent duration in stock 2 at time $t^1_{N^1(t)-1}$

2.2 Extended ACD model

To capture the dependence between durations in stocks we suggest that the ACD model be extended by adding durations from stocks beyond the focused one. The available durations for the focused stock are past finished durations in the same and the other stocks, and the length of the most recent incomplete duration in other stocks. For the presentation of the model we introduce a counting process and then proceed with the suggested formulation of the model.

Let the number of events for the kth duration series by time t be denoted by $N^k(t)$ and $t_0^k < t_1^k < t_2^k < \cdots < t_i^k$ the transaction times for the kth series. The most recent transaction time is then at $t^k_{N^k(t)}$ and the most recent duration is $d^k_{N^k(t)} = t^k_{N^k(t)} - t^k_{N^k(t)-1}$.

An example with two stocks, 1 and 2, is presented in Fig. 1. The expected length of the next duration $d^1_{N^1(t)}$ is conditioned at $t^1_{N^1(t)-1}$ in stock 1. The $d^1_{N^1(t)}$ may be conditioned on its own past durations, e.g., $d^1_{N^1(t)-1}, d^1_{N^1(t)-2}, \ldots, d^1_{N^1(t)-q}$, and other explanatory variables. If durations from a second stock are added to the conditional mean function, e.g., $d^2_{N^2(t)}, d^2_{N^2(t)-1}$ and $d^2_{N^2(t)-2}$, the most recent added duration $d^2_{N^2(t)}$ in stock 2 may not be completed when the conditioning takes place in stock 1. The observed length of the most recent duration $d^2_{N^2(t)}$ in stock 2 is τ_1, where $\tau_1 = t^1_{N^1(t)-1} - t^2_{N^2(t)-1}$ or $\tau_1 = \sum_{k=1}^{N^1(t)} d^1_{k-1} - \sum_{k=1}^{N^2(t)} d^2_{k-1}$, and $\tau_1 \geq 0$.

The proposed, extended ACD model is in a bivariate case for stocks 1 and 2:

$$\theta^1_{N^1(t)} = \omega^1 + \sum_{j=1}^{p_1} \alpha_j^1 d^1_{N^1(t)-j} + \sum_{j=1}^{q_1} \beta_j^1 \theta^1_{N^1(t)-j} + \pi' x^1_{N^1(t)-1} + \mathrm{e}^{\delta_0^1 \tau_1} \left(1 + \sum_{j=1}^{r_1} \delta_j^1 d^2_{N^2(t)-j} \right), \tag{4}$$

$$\theta^2_{N^2(t)} = \omega^2 + \sum_{j=1}^{p_2} \alpha^2_j d^2_{N^2(t)-j} + \sum_{j=1}^{q_2} \beta^2_j \theta^2_{N^2(t)-j} + \pi' x^2_{N^2(t)-1}$$
$$+ \mathrm{e}^{\delta^2_0 \tau_2} \left(1 + \sum_{j=1}^{r_2} \delta^2_j d^1_{N^1(t)-j} \right). \tag{5}$$

In (4) where completed durations from a stock 2 are added with r_1 lags, completed durations are given weights dependent on the size of τ_1, i.e., depending on how far away in time the completed durations are. The intuition behind the parameterization is that a large value of τ_1 in $\exp(\delta^1_0 \tau_1)$ gives low weight to the finished durations from stock 2 and a small value of τ_1 gives a larger weight to the finished durations when $\delta^1_0 < 0$. The conditional mean function (5) for stock 2 with durations from stock 1 added is formulated in a analogous manner.

When durations from a second stock are added to the conditional mean function as suggested the crucial point is the updating of the mean function. For the most recent duration from stock 2 to enter the mean function (4) a transaction in stock 1 must occur. During a long period without a transaction in stock 1 but with transactions in stock 2 information from stock 2 may not be included in the mean function for stock 1.

Obviously, the specifications of the final terms in (4) and (5) could take on other forms. For instance, the final term could be additive with $\theta^2_{N^2(t)-j}$, $j \geqslant 1$, and τ, entering (4) in a linear way and vice versa.

The conditional moment functions in (4) and (5) are easy to interpret with respect to effects of changes in post durations, etc. Obtaining the unconditional mean and variance is, however, quite difficult as it involves taking the expectation of the nonlinear final term in (4) and (5). To illustrate, consider the properties of an ACD(1,1) with one lag of the added duration. The model can be written

$$d^1_{N^1(t)} = \theta^1_{N^1(t)} \epsilon^1_{N^1(t)}, \quad \theta^1_{N^1(t)} = \omega^1 + \alpha^1 d^1_{N^1(t)-1} + \beta^1 \theta^1_{N^1(t)-1}$$
$$+ \mathrm{e}^{\delta^1_0 \tau_1} \left(1 + \delta^1_1 d^2_{N^2(t)-1} \right), \tag{6}$$

where $\epsilon^1_{N^1(t)}$ follows a standard exponential distribution. By taking expectations of both sides of (6), assuming that $d^1_{N^1(t)}$ and $\theta^1_{N^1(t)}$ are weakly stationary the unconditional mean may be expressed as

$$\mu_1 = E\left(\theta^1_{N^1(t)}\right) = E\left(d^1_{N^1(t)}\right) = \frac{\omega^1 + \varphi^1}{1 - \alpha^1 - \beta^1},$$

where $\varphi^1 = E\left[e^{\delta^1_0 \tau_1} \left(1 + \delta^1_1 \theta^2_{N^2(t)}\right)\right]$ can be either positive or negative depending on the sign of δ^1_1. To have positive durations the most natural conditions are $\alpha^1 + \beta^1 < 1$ and $\omega^1 > -\varphi^1$. Obtaining an explicit formulation of φ^1 and the variance appears more difficult than illuminating, and hence detailed expressions are not given.

One useful extension of the original ACD model that ensures positive expected durations is the log-ACD model by Bauwens and Giot (2000). The conditional mean function for a log-ACD(p,q) with explanatory variables is then

$$\theta^1_{N^1(t)} = \exp\left[\omega^1 + \sum_{j=1}^{p_1} \alpha^1_j d^1_{N^1(t)-j} + \sum_{j=1}^{q_1} \beta^1_j \ln \theta^1_{N^1(t)-j} \right.$$
$$\left. + \pi' x^1 + \mathrm{e}^{\delta^1_0 \tau_1} \left(1 + \sum_{j=1}^{r_1} \delta^1_j d^2_{N^2(t)-j}\right)\right]. \tag{7}$$

For the specification log-ACD(1,1) the only parameter restriction is $\beta^1 < 1$ and $\delta^1_0 < 0$.

2.3 Estimation

Engle and Russell (1998) popularized the quasi maximum likelihood (QML) estimator building on the exponential distribution for the estimation of the unknown parameters of the ACD model. By extending the conditioning set to also include observed durations in the other duration sequence, the same QML estimator can be used for our purpose. The estimator maximizes the log-likelihood function,

$$\ln \ell = -\sum_{j=1}^{T} \left[\ln \theta^k_j + \frac{d^k_j}{\theta^k_j}\right]. \tag{8}$$

The QML estimator is consistent when the conditional mean function is correctly specified. A correctly specified distribution may, however, render a more efficient estimator. For the practical estimation of the parameters we employ the log-ACD specification in (7), which we implement in the RATS package using the BFGS algorithm. The standard errors of the parameter estimates are the robust standard errors given by RATS.

Other estimators for the unknown parameters may also be considered, e.g., conditional least squares (CLS) or generalized method of moments (GMM). With these estimators the distribution assumption on the conditional duration is relaxed, though, relaxing the distributional assumption is not a major drawback given that the density is in the exponential family and θ^k_j is correctly specified (Bauwens et al. 2004).

Starting with the CLS estimator the prediction error from (4) to (5) is

$$e^k_{N^k(t)} = d^k_{N^k(t)} - \theta^k_{N^k(t)}.$$

The CLS estimator of the parameters in the vector ψ' minimizes the sum of the squared prediction errors

$$Q(\psi) = e'e,$$

where $e' = (e'_1, e'_2)$. Standard errors of the estimated parameters are obtained from the robust error option in RATS. A major benefit of leaving the QML based on a univariate exponential model is that joint estimation of the two conditional mean functions is feasible. By this it is easier to extend previous conditional mean functions to, e.g., be functions of lagged conditional means from other duration sequences. Naturally, if considering the joint estimation of the conditional mean functions the irregular intervals and updating of the conditional mean functions must be carefully considered.

The GMM estimator introduced by Hansen (1982) may also be considered for our purpose by utilizing the orthogonality conditions suggested by Grammig and Wellner (2002) and by potentially adding a parametrization for the covariation between duration sequences.

3 Data and descriptive statistics

The transaction data were downloaded from Ecovision, a provider of real time financial information from the Stockholm Stock Exchange. Four stocks, Nordea (NDA), SEB, Föreningsspabanken (FSB) and Handelsbanken (SHB), were recorded for 98 trading days (May 3 to September 30, 2005). The companies are active in the same line of business, the banking sector. Every transaction is recorded on a second scale with associated volume, bid, ask, and price variables. Descriptive statistics of durations of the four stocks are presented in Table 1.

The stocks are four of the most traded stocks on the Stockholm Stock Exchange and roughly of the same size. The number of observations indicates that the most frequently traded stock is NDA followed by SEB, SHB and FSB. Also the daily turnover indicates that NDA is the most traded stocks followed by the other three stocks. The tick size, i.e., the minimum amount a price can move, is for FSB, SEB and SHB 0.5 SEK and for NDA 0.25 SEK. The durations show strong serial correlation. The Ljung-Box statistics presented in Table 1 are high for all four stocks. Figure 2 shows the autocorrelation functions for the series. The autocorrelation functions share the same pattern, quite small and the decay is also quite slow.

Transaction data often show strong intraday seasonality patterns (e.g., Bauwens et al. 2002; Engle and Russell 1998). Figure 2 shows the intraday seasonality pattern for the duration of the current sample. Durations tend to be shorter when the market opens and near the closing time for all stocks. Similar patterns are also present in variables such as volume, spread and trade intensity. To account for the potential seasonality of the data the adjusted duration (Engle and Russell 1998) is computed as $d_i = D_i/\phi_i$, where D_i is the duration from the dataset and ϕ_i is a cubic spline with nodes on each hour. The adjusted duration d_i is used for estimation. In a

Table 1 Summary statistics for Nordea, SEB, FSB and SHB durations

	Mean	Std	Nr obs	LB_{100}	Min	Max	Daily turnover million SEK
NDA	35.1	70.9	73,723	29,614	0	1,415	586.9
SEB	45.7	92.9	56,590	11,763	0	1,767	326.4
FSB	58.9	115.6	44,083	7,106	0	2,270	298.9
SHB	50.6	99.5	51,094	15,454	0	1,606	343.4

LB_{100} is the Ljung-Box autocorrelation statistic for 100 lags

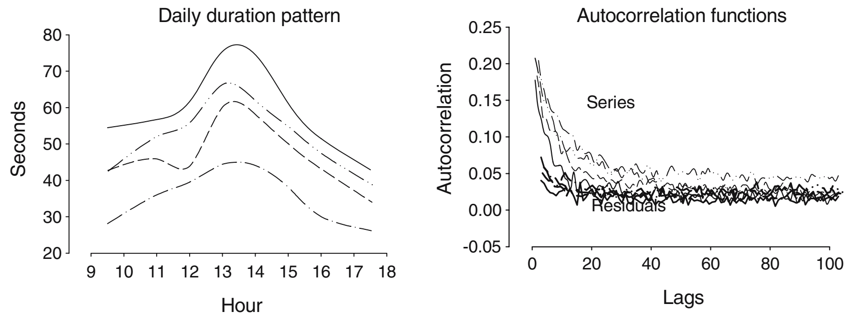

Fig. 2 Average duration lengths per hour smoothed with a cubic spline (left) and autocorrelation functions of durations and residuals *(right)*. In the figure are FSB (*solid line*), SHB (*dashed dot dot line*), SEB (*dashed line*) and NDA (*dashed dot line*)

similar way the other variables, volume, spread, trade intensity and the added durations are adjusted to account for the diurnal effect.

There are many consecutive zero durations, i.e., multiple transactions within a second, in the data. Suggestions in the literature of the treatment of zero durations are, e.g., aggregate all data within a second (cf. Engle and Russell 1998) or assume that the distance between transactions within a second are equally spaced (cf. Darolles and Gouriéroux 2000). In this paper the zero durations are replaced with 0.5.

In the empirical part volume and spread changes are used as explanatory variables together with a measure of trade intensity. A variable capturing the intensity of trade (Engle and Russell 1998) can be constructed by dividing the number of transactions within a price duration with the length of a price duration. Bauwens and Giot (2000) use a similar approach using instead the spread duration. A price duration is calculated as the time it takes for the price to move a predetermined number of ticks. Obviously a new price duration is initiated more often with a low predetermined tick size. The number of ticks is here chosen as 1. Other studies, e.g., Engle and Russell (1998) suggests the number of ticks as 4. The price used for the price duration is defined as the midprice, i.e., the price in the middle of bid and ask at time i, when a transaction occur.

4 Results

The results to be reported are based on the log-ACD model. A main reason for using the log-ACD in favour of the ACD model is that numerical problems were faced when a pure ACD model was implemented. To find a lag structure we employ the AIC criterion. The minimized AIC gives us the model specification.

Tables 2–5 give the CLS and QML estimation results for the stocks with differences in volume, spread, trade intensity and durations as explanatory variables. The parameter estimates for the explanatory variable volume change are significant for NDA but not for SHB, SEB and FSB. The sign of the volume change variable are throughout positive. For all series the spread change has a significant and negative effect. Trade intensity changes have significant effects for FSB, SEB with added durations from NDA but not for the models of SHB and NDA. The sign is positive

Table 2 Estimates of Nordea (NDA) with explanatory variables and durations from SEB A, Föreningssparbanken (FSB) and Handelsbanken (SHB) (Robust t statistics in parenthesis)

Variable		NDA			
	NDA	SEB A	FSB	SHB	CLS
d_{t-1}	0.158	0.157	0.157	0.157	0.041
	(37.67)	(41.66)	(40.86)	(43.62)	(7.61)
d_{t-2}	−0.023	−0.021	−0.021	−0.022	0.0003
	(−4.27)	(−4.31)	(−4.02)	(−4.70)	(0.04)
d_{t-3}	−0.014	−0.012	−0.013	−0.013	−0.004
	(−2.69)	(−2.43)	(−2.48)	(−2.60)	(−0.77)
d_{t-4}	−0.010	−0.009	−0.007	−0.010	−0.001
	(−2.29)	(−2.17)	(−1.77)	(−2.50)	(−0.27)
θ_{t-1}	0.864	0.856	0.856	0.861	0.91
	(130.78)	(121.8)	(131.3)	(142.2)	(96.2)
Volume change	0.097	0.090	0.103	0.097	0.016
	(2.49)	(3.12)	(2.63)	(2.37)	(0.21)
Trade intensity change	0.007	0.007	0.006	0.007	0.003
	(1.36)	(1.61)	(1.88)	(1.51)	(1.44)
Spread change	−0.326	−0.329	−0.316	−0.328	−0.588
	(−5.71)	(−5.83)	(−22.97)	(−5.73)	(−14.0)
δ_0		0.002	0.007	0.003	
		(1.18)	(4.62)	(1.81)	
δ_1		0.002	0.005	0.002	
		(2.02)	(3.88)	(2.08)	
δ_2		0.001	−0.003	0.002	
		(1.32)	(−1.91)	(1.87)	
δ_3		0.003	0.005	0.005	
		(6.07)	(4.29)	(4.22)	
δ_4		0.0001			
		(0.14)			
δ_5		0.002			
		(1.79)			
δ_6		0.0009			
		(1.28)			
δ_7		0.004			
		(7.57)			
δ_8		0.002			
		(2.30)			
δ_9		0.001			
		(1.93)			
Constant	−0.140	−1.163	−1.16	−1.153	−0.039
	(−21.60)	(−148.6)	(−174.0)	(−185.9)	(−11.8)
LB_{100}	919.2	941.7	947.1	924.2	5499.1
LB^2_{100}	114.3	111.2	117.5	110.7	154.9
$\ln l$	−58717.8	−58492.0	−58538.8	−58599.9	

LB_{100} is the Ljung-Box statistic of the residuals over 100 lags. LB^2_{100} is the same statistic for squared residuals. Residuals are calculated as $(d_{N(t)} - \widehat{\theta}_{N(t)})/\widehat{\theta}_{N(t)}$

except for SHB. The interpretation of a negative parameter sign is that a positive change in the variable shortens the next duration. Brännäs and Simonsen (2006) who also utilize change variables when studying Ericsson B, for a different sample, also find a negative effect of the spread change and a positive one for the volume change.

Table 3 Estimates of Föreningssparbanken (FSB) with explanatory variables and durations from SEB A, Nordea (NDA) and Handelsbanken (SHB) (Robust t statistics in parenthesis)

Variable		FSB			
	FSB	SEB A	NDA	SHB	CLS
d_{t-1}	0.140	0.140	0.140	0.140	0.053
	(25.46)	(30.13)	(26.50)	(31.84)	(16.92)
d_{t-2}	−0.046	−0.032	−0.046	−0.046	−0.015
	(−7.23)	(−2.56)	(−6.77)	(−7.89)	(−2.73)
d_{t-3}	−0.025	−0.015	−0.025	−0.025	0.001
	(−6.28)	(−1.56)	(−3.98)	(−4.09)	(0.10)
d_{t-4}	−0.023	−0.017	−0.023	−0.023	−0.011
	(−3.69)	(−2.03)	(−3.31)	(−3.40)	(−1.87)
d_{t-5}	0.003	0.007	0.003	0.003	0.001
	(0.26)	(0.94)	(0.45)	(0.33)	(0.21)
d_{t-6}	−0.013	−0.011	−0.013	−0.013	−0.006
	(−2.00)	(−1.67)	(−2.11)	(−2.16)	(−1.05)
d_{t-7}	−0.026	−0.008	−0.025	−0.025	−0.019
	(−5.58)	(−0.90)	(−4.78)	(−6.25)	(−4.71)
θ_{t-1}	0.988	0.888	0.985	0.985	0.990
	(297.5)	(11.3)	(138.2)	(151.5)	(363.9)
Volume change	0.001	0.047	0.001	0.003	0.011
	(0.05)	(1.03)	(0.09)	(0.18)	(0.64)
Trade intensity change	0.014	0.047	0.016	0.017	0.013
	(2.72)	(2.69)	(2.02)	(2.47)	(2.86)
Spread change	−0.133	−0.161	−0.134	−0.137	−0.375
	(−2.42)	(−2.79)	(−3.00)	(−2.30)	(−5.89)
δ_0		0.007	0.0009	0.000	
		(1.29)	(1.00)	(0.01)	
δ_1		0.004	−0.00009	0.0006	
		(1.07)	(−0.14)	(1.63)	
δ_2				0.0005	
				(1.18)	
Constant	−0.011	−1.085	−1.014	−1.014	−0.005
	(−4.15)	(−19.74)	(−166.0)	(−186.18)	(−3.67)
LB_{100}	199.0	194.7	194.5	197.3	976.1
LB^2_{100}	122.1	117.8	103.1	106.0	135.5
$\ln l$	−39504.8	−39469.7	−39492.5	−39481.2	

LB_{100} is the Ljung-Box statistic of the residuals over 100 lags. LB^2_{100} is the same statistic for squared residuals. Residuals are calculated as $(d_{N(t)} - \widehat{\theta}_{N(t)})/\widehat{\theta}_{N(t)}$

The estimated parameters α and β in Tables 2–5 are all significant with a few exceptions. Also exclusion of the insignificant parameters were considered. However, such specification is not minimizing the AIC. The parameter β is close to one for all stocks which is also what is found in other studies of transaction data (e.g., Brännäs and Simonsen 2006). The interpretation of a β parameter estimate close to one is that a short (long) duration tend to be followed by a short (long) duration.

Next, considering the added dependence terms in (4) and (5) we find significant parameter estimates for SEB, SHB and NDA, though not all, while added dependence terms to FSB are all insignificant. More specifically we find significant estimates of the first and third lags of added durations from SEB, FSB and SHB to NDA and also the eighth and ninth lag of durations from SEB (Table 2). In the estimates of added durations to SEB we find significant parameter estimates

Table 4 Estimates of SEB A with explanatory variables and durations from Föreningssparbanken (FSB), Nordea (NDA) and Handelsbanken (SHB) (Robust t statistics in parenthesis)

Variable		SEB			
	SEB	NDA	FSB	SHB	CLS
d_{t-1}	0.140	0.138	0.138	0.139	0.0459
	(33.99)	(36.63)	(38.49)	(37.57)	(11.07)
d_{t-2}	−0.017	−0.015	−0.015	−0.014	0.0011
	(−3.05)	(−2.58)	(−2.96)	(−2.89)	(0.21)
d_{t-3}	−0.023	−0.018	−0.019	−0.020	−0.0150
	(−4.31)	(−3.72)	(−3.75)	(−3.83)	(−3.25)
θ_{t-1}	0.845	0.825	0.832	0.829	0.9160
	(81.84)	(73.97)	(80.81)	(79.66)	(74.79)
Volume change	0.012	0.017	0.007	0.011	−0.092
	(0.20)	(0.31)	(0.13)	(0.20)	(−0.98)
Trade intensity change	0.013	0.014	0.013	0.012	0.003
	(2.02)	(2.24)	(1.60)	(1.56)	(0.47)
Spread change	−0.206	−0.201	−0.208	−0.195	−0.539
	(−3.73)	(−5.76)	(−3.81)	(−3.59)	(−6.31)
δ_0		−0.002	0.013	0.010	
		(−0.90)	(6.71)	(5.31)	
δ_1		0.002	0.003	−0.005	
		(1.36)	(2.28)	(−2.67)	
δ_2		0.007		0.004	
		(4.95)		(2.47)	
δ_3		0.004		0.002	
		(2.07)		(1.56)	
δ_4		0.007		0.004	
		(3.75)		(2.66)	
δ_5		0.003			
		(2.36)			
δ_6		0.003			
		(1.90)			
δ_7		0.005			
		(3.52)			
Constant	−0.120	−1.156	−1.143	−1.143	−0.034
	(−15.57)	(−113.8)	(−139.6)	(−140.7)	(−8.98)
LB_{100}	326.7	323.1	327.7	323.3	1905.4
LB^2_{100}	130.1	137.6	128.9	124.4	189.1
$\ln l$	−49242.9	−48960.0	−49097.7	−49096.4	

LB_{100} is the Ljung-Box statistic of the residuals over 100 lags. LB^2_{100} is the same statistic for squared residuals. Residuals are calculated as $(d_{N(t)} - \widehat{\theta}_{N(t)})/\widehat{\theta}_{N(t)}$

except for the first and sixth lag of added durations from NDA and the third lag of durations from SHB (Table 4). Added durations to SHB are significant for the first and fifth lags of FSB and fifth lag of NDA, while added durations from SEB are not (Table 5).

Regarding the parameter sign, the expected sign of lagged added dependence terms is positive. Positive signs of the estimates implies that long durations in the added duration series has a positive impact on the conditional mean function i.e., prolongs the next duration. Positive parameter signs is also found for a major part of the estimates. The expected sign of the δ_0 parameter is negative as this implies that the impact of the other stocks is decreasing with the size of τ. However, we

Table 5 Estimates of Handelsbanken (SHB) with explanatory variables and durations from Föreningssparbanken (FSB), Nordea (NDA) and SEB A (Robust t statistics in parenthesis)

Variable		SHB			
	SHB	NDA	SEB	FSB	CLS
d_{t-1}	0.143	0.143	0.143	0.1430	0.052
	(37.48)	(29.46)	(35.08)	(31.66)	(20.33)
d_{t-2}	−0.041	−0.041	−0.040	−0.040	−0.013
	(−10.30)	(−5.99)	(−6.50)	(−6.39)	(−3.08)
d_{t-3}	−0.028	−0.028	−0.027	−0.028	0.001
	(−6.08)	(−3.96)	(−8.37)	(−4.30)	(0.26)
d_{t-4}	−0.014	−0.014	−0.013	−0.014	−0.012
	(−2.70)	(−3.13)	(−2.24)	(−2.07)	(−4.12)
d_{t-5}	−0.007	−0.007	−0.008	−0.007	−0.001
	(−1.30)	(−1.32)	(−1.29)	(−1.16)	(−0.22)
d_{t-6}	−0.019	−0.020	−0.019	−0.019	−0.005
	(−3.73)	(−2.74)	(−3.45)	(−2.96)	(−0.93)
d_{t-7}	−0.023	−0.022	−0.020	−0.021	−0.016
	(−5.39)	(−4.44)	(−4.60)	(−4.39)	(−4.04)
θ_{t-1}	0.988	0.985	0.982	0.986	0.990
	(392.0)	(253.7)	(190.7)	(217.1)	(300.0)
Volume change	0.013	0.011	0.011	0.016	−0.016
	(0.68)	(0.54)	(0.48)	(0.78)	(−0.87)
Trade intensity change	−0.004	−0.004	−0.002	−0.004	−0.011
	(−0.89)	(−0.73)	(−0.33)	(−0.69)	(−1.62)
Spread change	−0.356	−0.362	−0.368	−0.365	−0.603
	(−4.62)	(−5.37)	(−5.12)	(−5.11)	(−8.79)
δ_0		0.0003	0.0012	0.0011	
		(0.63)	(1.80)	(1.96)	
δ_1		0.0005	0.0007	0.0001	
		(0.90)	(1.49)	(0.26)	
δ_2		−0.0001	0.0006	0.0008	
		(−0.15)	(1.50)	(2.21)	
δ_3		0.0002	0.0002	0.0003	
		(0.56)	(0.48)	(0.89)	
δ_4		−0.0001	0.0006	−0.0001	
		(−0.28)	(1.46)	(−0.30)	
δ_5		0.0011		−0.0008	
		(2.93)		(−2.63)	
δ_6		0.0006			
		(1.63)			
Constant	−0.0130	−1.0176	−1.0205	−1.0163	−0.006
	(−5.35)	(−248.9)	(−188.0)	(−219.2)	(−3.48)
LB_{100}	388.5	384.3	369.2	389.0	1844.8
LB^2_{100}	186.7	194.5	126.1	231.9	232.7
$\ln l$	−43826.75	−43716.1	−43746.6	−43760.6	

LB_{100} is the Ljung-Box statistic of the residuals over 100 lags. LB^2_{100} is the same statistic for squared residuals. Residuals are calculated as $(d_{N(t)} - \widehat{\theta}_{N(t)})/\widehat{\theta}_{N(t)}$

find the sign of δ_0 positive except for SEB with durations from NDA, although insignificant (Table 4).

To sum up, we find significant and the expected positive parameter estimates, though not for all lags, of the added durations to NDA and SEB. Also added durations to SHB are significant except durations from SEB. For the added durations to FSB all estimates are insignificant.

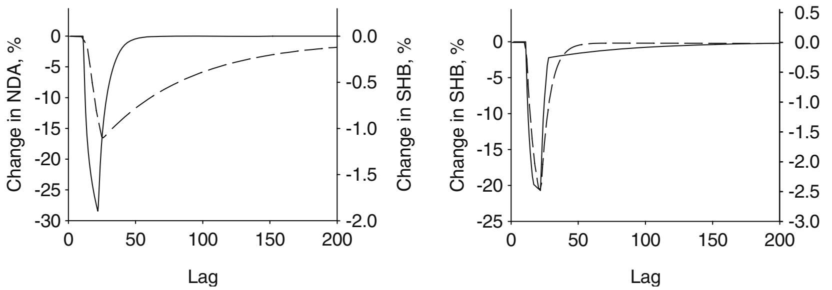

Fig. 3 The change in percent of a 50 percent shortened duration in 10 consequtive durations from the mean duration at time $t = 10$ in Nordea (*NDA*) (*left*) and Handelsbanken (*SHB*) (*right*). The solid line in the left figure shows the response in NDA of a shortened duration in NDA and in the right figure the response in SHB of a shortened duration in SHB. The *dashed lines* show the responses of a shorthened duration in the other stock

When considering the CLS estimator the parameter signs and sizes are roughly the same, though with a few exceptions. Some of the parameter signs change when using the CLS estimator, e.g., the second lag of lagged durations in NDA. The estimated parameters of volume change are negative for SEB and SHB. The serial correlation of the residuals is still high and increased for all stocks.

One may use a Granger causality test (Granger 1969) to examine if the inclusion of a duration from other stocks to the conditional mean in (4) and (5) adds explanatory power or not. Considering individual parameter estimates we find significant parameters of adding durations from stocks beyond the focused one to NDA and SEB but not for FSB. Individual parameters of added durations to SHB are significant for NDA and FSB but not for SEB. Consequently it is found that NDA granger causes SEB and SEB granger causes NDA. FSB Granger causes NDA, SEB and SHB, while NDA, SEB SHB is not Granger causing FSB. FSB and NDA Granger causes SHB while SEB is not. Also a likelihood ratio test is applied to test if the individual parameters of the added durations are jointly, significantly different from zero (Tables 2–5). The test statistic has in large sample a χ^2 distribution with the degrees of freedom equal to the number of additional parameters, i.e., the number of added lagged durations including δ_0. The 5 percent critical value range from 5.99 with 2 degrees of freedom to 18.31 with 10 degrees of freedom. It is found that there is Granger causality between all the stocks, including FSB and SHB with added durations from SEB.[1] Similar results have also been found for other stocks and markets, e.g., Spierdijk et al. (2002) and Bauwens and Hautsch (2004).[2]

In the models reported in Tables 2–5 there is serial correlation left that we were unable to reduce further. To test for remaining serial correlation the Ljung-Box statistic is used. The test statistic has asymptotically a χ^2_{100} distribution and a 5 percent critical value of 124.3. For the residuals of NDA, SEB, FSB and SHB the null of no serial correlation is rejected for all models. For the squared residuals the

[1] The result is also valid for a test statistic with 1 percent critical values.

[2] In an early draft of the paper, from another sample, we found that AstraZeneca is not Granger causing Ericsson B, while Ericsson B Granger causes AstraZeneca.

null hypothesis is rejected for all models of SHB and NDA. Note that the reported results are based on deseasonalized data.

The estimated models may be used to evaluate and illustrate the response to a chock in, e.g., the added duration. Figure 3 gives the response in the expected mean duration $\theta_{N(t)}$ to a 50 percent reduction of 10 consecutive durations for SHB and NDA illustrated. The shocks have a small and diminishing effect on $\theta_{N(t)}$. Hence, the estimated models are stationary. The response in percent in NDA of a shortened duration in SHB is roughly twice the size of the response in SHB of a shortened duration is NDA.

5 Conclusions

The paper proposed adding durations from a second stock to the conditional mean function in the ACD model of Engle and Russell (1998). By including durations from a second stock dependence between duration series is captured in the model. The model is applied to four stocks, Nordea, Fö reningssparbanken, SEB and Handelsbanken, all from the banking sector, traded on the Stockholm Stock Exchange.

In the empirical part, we find Granger causality between all the stocks, Nordea, Föreningssparbanken, SEB and Handelsbanken, when the added durations from stocks beyond the focused one is considered jointly. For the individual parameter estimates we find Granger causality between Nordea and SEB. Föreningssparbanken Granger causes Nordea, SEB and Handelsbanken, while Nordea, SEB and Handelsbanken is not Granger causing Fö reningssparbanken. Handelsbanken Granger causes Nordea and SEB. Nordea and F öreningssparbanken are Granger causing Handelsbanken while SEB is not. This result indicates that there may be duration dependence between stocks active in the banking sector on the Stockholm Stock Exchange. The dependence may be caused by new information revealed by the trade intensity. In view of this empirical result the suggested model extension is able of capturing dependence between duration series and of providing an improvement in the econometric specification of the model. The attraction of the extension of the model is the simplicity by which information from other duration series can be added to the ACD model.

The parameter estimates of spread change is significant for all stocks and volume change is significant for Nordea. The expected negative parameter estimates are found for spread change and trade intensity change for Handelsbanken. The descriptive statistics of duration and other variables show presence of time of day seasonality in the stocks from the Stockholm Stock Exchange, cf. the results for other stock markets.

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